ROBOTYKA ćwiczenia 11.10.2019 r.

10)

$$f = k_1 \sin(q_2) - k_2 q_1 q_2^2$$

$$\frac{df}{dt} = k_1 \cos(q_2) q_2^? - (k_2 q_2^2 + 2k_2 q_1 q_2)$$

LISTA 2

1)

$$i, \; j = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad k = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\langle j, k \rangle = 0 \cdot \frac{1}{\sqrt{2}} + 1 \cdot 0 + 0 \cdot \left(-\frac{1}{\sqrt{2}}\right) = 0$$

$$|k| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$$

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$j \times k = \begin{vmatrix} u & v & w \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{vmatrix} = -\frac{1}{\sqrt{2}} u - \frac{1}{\sqrt{2}} w = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Unormowany → długość wektora = 1

3)

2) $i = (i_x, i_y, i_z)^T$

$i_x = \frac{2}{3}$

$i_y = ?$

$i_z = 0$

$1 = \langle i, i \rangle = \frac{2}{3} \cdot \frac{2}{3} + i_y^2$

$\frac{5}{9} = i_y^2$

$i_y = \frac{\sqrt{5}}{3}$

$j = (0, 0, 1)^T$

$$i \times j = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} \frac{2}{3} & \frac{\sqrt{5}}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} = \left(\frac{\sqrt{5}}{3}, \frac{2}{3}, 0\right)^T = k$$

3)

$$\det \left[i_0 \,\middle|\, j_0 \,\middle|\, k_0 \right] =$$

$$= \det \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix} = 1 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$-\left(-\frac{1}{2} \cdot \frac{1}{2} \cdot 1\right) = \frac{3}{4} + \frac{1}{4} = 1$$

$$\det \left[i_1 \,\middle|\, j_1 \,\middle|\, k_1 \right] = \det \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= -1 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}} \cdot (-1) \cdot \frac{1}{\sqrt{2}} \right) =$$

$$= -\frac{1}{2} - \frac{1}{2} = -1$$

$$i_1 = (0, 1, 0)^T$$

$$R_0^1 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} i_0 j_1 & j_1 i_0 & k_1 i_0 \\ i_1 j_0 & j_1 j_0 & k_1 j_0 \\ i_1 k_0 & j_1 k_0 & k_1 k_0 \end{bmatrix}$$

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$$p_1 = R^0_1 \, p_0$$

$$R^0_1 = \left(R^1_0\right)^T$$

$$p_1 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1-3\sqrt{3}}{2} \\ \frac{7-\sqrt{3}}{2\sqrt{2}} \\ \frac{1-\sqrt{2}}{2\sqrt{2}} \end{bmatrix}$$

Alcune rotazioni

$$\mathrm{rot}(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix}$$

$$\mathrm{rot}(y,\beta) = \begin{bmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix}$$

$$\mathrm{rot}(z,\gamma) = \begin{bmatrix} C_\gamma & -S_\gamma & 0 \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5) $\mathrm{rot}(x,\alpha) \cdot \mathrm{rot}(x,\beta) =$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\beta & -S_\beta \\ 0 & S_\beta & C_\beta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha C_\beta + S_\alpha S_\beta & -C_\alpha S_\beta - S_\alpha C_\beta \\ 0 & S_\alpha C_\beta + C_\alpha S_\beta & -S_\alpha C_\beta + C_\alpha C_\beta \end{bmatrix}$$

*

$$rot(x,\beta)\cdot rot(x,\alpha) =$$

$$= \ldots$$

$$rot(x,\alpha)\cdot rot(y,\beta) =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{bmatrix} \begin{bmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix} =$$

$$= \begin{bmatrix} C_\beta & 0 & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$rot(y,\beta)\cdot rot(x,\alpha) = \begin{bmatrix} C_\beta & S_\alpha S_\beta & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{\alpha\beta} & -S_{\alpha\beta} \\ 0 & S_{\alpha\beta} & C_{\alpha\beta} \end{bmatrix}$$

Dla Wdisi jednej osi to suma kątów !

6) $RPY\left(-\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{2}\right) =$

$$= rot\left(z, -\frac{\pi}{2}\right) \cdot rot\left(y, \frac{\pi}{4}\right) \cdot rot\left(x, \frac{\pi}{2}\right)$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

$P \rightarrow$ kolumny

$L \rightarrow$ wiersze

$$= \begin{matrix} C_\alpha C_\beta \\ S_\alpha C_\beta \\ -S_\beta \end{matrix}$$

8) $r_{31} = -S_\beta \Rightarrow \beta \Rightarrow$ $\quad$ $\beta = -\arcsin(r_{31})$

$\downarrow$

wybieramy 1 opcje

$$\left[\begin{matrix} r_{21} = S_\alpha C_\beta \\ r_{11} = C_\alpha C_\beta \end{matrix} \Rightarrow \alpha \right]$$

$$\left[\begin{matrix} r_{32} = C_\beta S_\gamma \\ r_{33} = C_\beta S_\gamma \end{matrix} \Rightarrow \gamma \right]$$