ROBOTYKA (1)

[C] 25.10.2019r.

LISTA 4

1) $Z = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \in SE(3)$

$Z^{-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$Z \cdot Z^{-1} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = J_4 = \begin{bmatrix} RA + TC & RB + TD \\ C & D \end{bmatrix}$

$= \begin{bmatrix} J_3 & 0 \\ 0 & 1 \end{bmatrix}$

$D = 1$
$C = 0_{1\times 3}$

$R \cdot A = J_3 \Rightarrow A = R^{-1} = R^T$

$RB = -T \quad / \cdot R^{-1}$

$R^T R B \Rightarrow B = -R^T T$

$Z^{-1} = \begin{bmatrix} R^T & -R^T T \\ 0 & 1 \end{bmatrix}$

2) $R = rot(z, \psi)$ AR $A = [e_1] \vee A = [e_2] \vee A = [e_3]$

$rot(z,\psi) = \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\frac{\partial rot(z,\psi)}{\partial \psi} = \begin{bmatrix} -S\psi & -C\psi & 0 \\ C\psi & -S\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} =$ ZGADUJEMY

$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = [e_3]\, rot(z,\psi)$

$[e_3] = [0, 0, 1]$

3) $R = RPY(\phi, \theta, \psi) = rot(z, \phi) \cdot rot(y, \theta) \cdot rot(x, \psi)$
$= rot_{z,\phi} \cdot rot_{y,\theta} \cdot rot(x, \psi)$

$[\omega_s] = \dot{R} R^T \qquad [\omega_b] = R^T \dot{R}$

$\dot{R} = \dot{rot}_z \cdot rot_y \cdot rot_x + rot_z \cdot \dot{rot}_y \, rot_x + rot_z \, rot_y \, \dot{rot}_x$

$R^T = rot_x^T \cdot rot_y^T \cdot rot_z^T$ (rotacja transponowana)
$(ABC)^T = C^T B^T A^T$

$[\omega_s] = \dot{R} \cdot R^T =$
(usuwamy macierze jednostkowe)

$= \dot{rot}_z \underbrace{rot_y \, rot_y^T}_{I_3} \cdot rot_z^T + rot_z \cdot \dot{rot}_y \cdot rot_y^T \cdot rot_z^T +$

$[e_1] \, rot(x, \psi)$

$+ \, rot_z \, rot_y \, \dot{rot}_x \cdot rot_x^T \, rot_y^T \, rot_z^T$

$$\frac{d}{dt} rot(y, \theta(t)) = \frac{\partial \, rot(y, \theta)}{\partial \theta} \cdot \frac{d\theta}{dt} =$$

$$\frac{d\theta}{dt} = \dot{\theta}$$

$$= \dot{\theta} \frac{\partial \, rot(y, \theta)}{\partial \theta} = [e_2] \cdot rot(y, \theta) \cdot \dot{\theta}$$

I tak każdy

$[\omega_s] = \dot{\phi} [e_3] + \dot{\theta} \, rot_z [e_2] \cdot rot_z^T + \dot{\psi} \, rot_z \cdot rot_y [e_1] \, rot_y^T \cdot rot_z^T$

$$\mathrm{rot}(z,\varphi)[e_2]\,\mathrm{rot}(z,-\varphi) = \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \mathrm{rot}(z,-\varphi)$$

$$= \begin{bmatrix} 0 & 0 & C\varphi \\ 0 & 0 & S\varphi \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C\varphi & S\varphi & 0 \\ -S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & C\varphi \\ 0 & 0 & S\varphi \\ C\varphi & -S\varphi & 0 \end{bmatrix}$$

$$B = \mathrm{rot}(z,\varphi)\,\mathrm{rot}(y,\theta) = \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

$$= \begin{bmatrix} C\varphi C\theta & -S\varphi & C\varphi S\theta \\ S\varphi C\theta & C\varphi & S\varphi S\theta \\ -S\theta & 0 & C\theta \end{bmatrix} \text{①}$$

$$B\cdot[e_1] = \text{①} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & C\varphi S\theta & S\varphi \\ 0 & S\varphi S\theta & -C\varphi \\ 0 & C\theta & 0 \end{bmatrix} \text{②}$$

$$B\cdot[e_1]\cdot B^T = \text{②} \cdot \begin{bmatrix} C\varphi C\theta & S\varphi C\theta & -S\theta \\ -S\varphi & C\varphi & 0 \\ C\varphi S\theta & S\varphi S\theta & C\theta \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & S\theta & S\varphi C\theta \\ -S\theta & 0 & -C\varphi C\theta \\ -S\varphi C\theta & C\varphi C\theta & 0 \end{bmatrix}$$

4) (druga)

$$[\omega_s] = \dot{\varphi}\cdot\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \dot{\theta}\begin{pmatrix} -S\varphi \\ C\varphi \\ 0 \end{pmatrix} + \dot{\psi}\begin{pmatrix} C\varphi C\theta \\ S\varphi C\theta \\ -S\theta \end{pmatrix}$$

$$\omega_s = \begin{pmatrix} \omega_{s1} \\ \omega_{s2} \\ \omega_{s3} \end{pmatrix} = \begin{bmatrix} 0 & -S\varphi & C\varphi C\theta \\ 0 & C\varphi & S\varphi C\theta \\ 1 & 0 & -S\theta \end{bmatrix} \begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

5)

$$\underset{6\times 6}{v_1} = \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} v_b$$

$$B = \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} A & E \\ C & D \end{bmatrix}$$

$$B^{-1} \cdot B = I_6$$

$$B^{-1} \cdot B = \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} R & [T]R \\ 0 & R \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix}$$

$$A \cdot R + E0 = I_3$$

$$AR = I_3$$

$$A = R^T$$

$$A \cdot [T]R + E \cdot R = 0$$

$$R^T [T] \cdot R + ER = 0$$

$$ER = -R^T [T] \cdot R \quad / \cdot R^T$$

$$E = -R^T [T]$$

$$CR + D0 = 0$$

$$CR = 0$$

$$C = 0$$

$$\boxed{(0,0,0)^T \quad \begin{matrix} 0 \\ 6 \\ 6 \end{matrix}}$$

$$C \cdot [T]R + DR = I_3$$

$$D = R^T$$

$$B^{-1} = \begin{bmatrix} R^T & -R^T[T] \\ 0 & R^T \end{bmatrix}$$

6) Czy macierz obrotu? 4 wiersz → 0, 0, 0, 1

$$A = \begin{bmatrix} C\alpha & 0 & S\alpha & 2 \\ 0 & -1 & 0 & 0 \\ S\alpha & 0 & -C\alpha & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A \cdot A^{-1} = I_4$$

$\cancel{A = R^T}$

$$R = R^T = \begin{bmatrix} C\alpha & 0 & S\alpha \\ 0 & -1 & 0 \\ S\alpha & 0 & -C\alpha \end{bmatrix}$$

$$\det R = C\alpha^2 + S\alpha^2 = 1$$

$$R \cdot R^T = R^T R = \begin{bmatrix} C\alpha & 0 & S\alpha \\ 0 & -1 & 0 \\ S\alpha & 0 & -S\alpha \end{bmatrix} \begin{bmatrix} C\alpha & 0 & S\alpha \\ 0 & -1 & 0 \\ S\alpha & 0 & -C\alpha \end{bmatrix} =$$

$$= \begin{bmatrix} C\alpha^2 + S\alpha^2 & 0 & C\alpha S\alpha - S\alpha S\alpha \\ 0 & 1 & 0 \\ S\alpha C\alpha - S\alpha C\alpha & 0 & S\alpha^2 + C\alpha^2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$