ROBOTYKA (1)

[W] 16. 01. 2017r.

kinematyka

$$q \xrightarrow{\text{prosta}} k(q) = x$$

odwrotna

dynamika

$$u = Q(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q)$$

$q(t)$ - będzie zależeć od warunków początkowych

w tym przypadku:

$q(0)$

$\dot{q}(0)$

$u(\cdot)$
+ warunki początkowe $\Rightarrow$ $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$, $\vdots$ $\quad q(\cdot)$

odwrotne

$u(\cdot)$

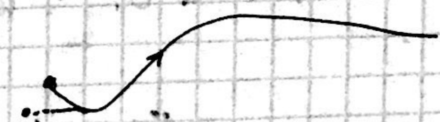
odwrotne

$q(\cdot) \in C^2$

$$Q(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q) = u$$

$+ \; q_d(\cdot) \in C^2$

q_d — desired



Q

METODA OBLICZANEGO MOMENTU (CT)

1° Linearyzacja modelu - wprowadzamy nowe sterowanie (v)

$$Q(q)v + C(q,\dot{q})\dot{q} + D(q) = u$$

$$Q(q)\ddot{q} = Q(q)v \qquad / Q^{-1}$$

$$\ddot{q} = v$$

nieosobliwa = odwracalna

2° Zamknięcie pętli sprzężenia zwrotnego

Błąd: $e(\cdot) = q(\cdot) - q_d(\cdot)$

$$v = \ddot{q}_d - \underbrace{K_d(\dot{q} - \dot{q}_d) - K_p(q - q_d)}_{\substack{\text{wsp.} \\ \text{stałe macierzowe} \\ \text{(dobierany)}}}$$

$$\ddot{e} + K_d\dot{e} + K_p e = 0$$

$$\begin{bmatrix} \ddot{e}_1 \\ \vdots \\ \ddot{e}_n \end{bmatrix} + \begin{bmatrix} K_d^1 & & 0 \\ & \ddots & \\ 0 & & K_d^n \end{bmatrix} \begin{bmatrix} \dot{e}_1 \\ \vdots \\ \dot{e}_n \end{bmatrix} + \begin{bmatrix} K_p^1 & & 0 \\ & \ddots & \\ 0 & & K_p^n \end{bmatrix} \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

K_p, K_d - macierze diagonalne

$$\ddot{e}_i + K_d^i \dot{e}_i + K_p^i e_i = 0 \qquad i = 1, \ldots, n$$

$$\ddot{e} + K_d\dot{e} + K_p e = 0 \qquad \rightarrow \text{równanie skalarne}$$

$$e(t) \xrightarrow{t \to \infty} 0$$

Jak dobrać K_p, K_d ?

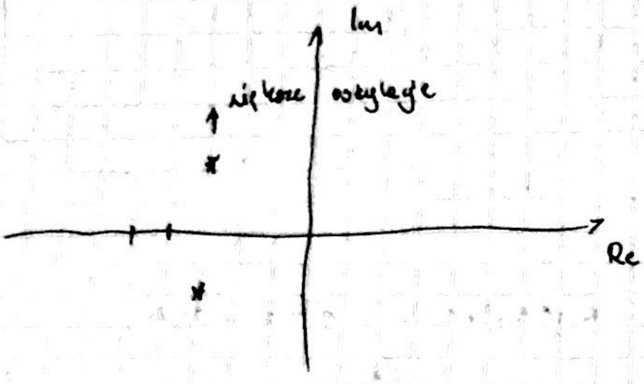
→ wierszanie charakterystyczne

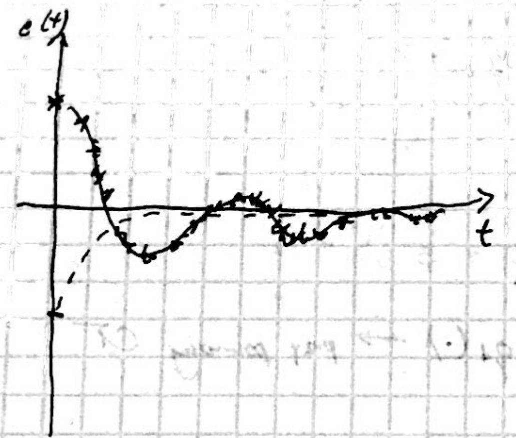
$$s^2 + K_d s + K_p = 0$$

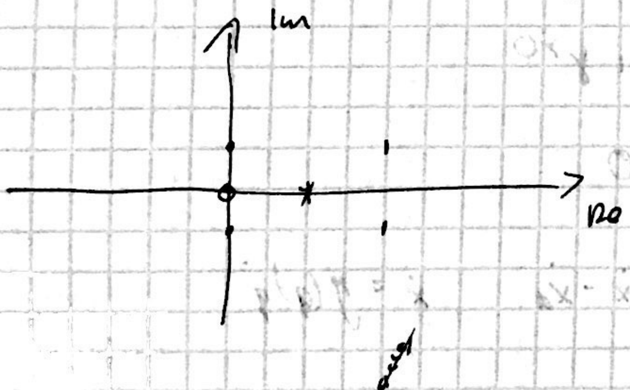
→ pierwiastki równania charakterystycznego

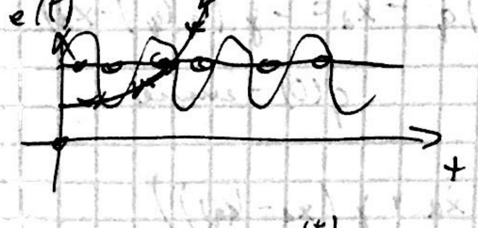
$$\Delta = k_d^2 - 4k_p$$

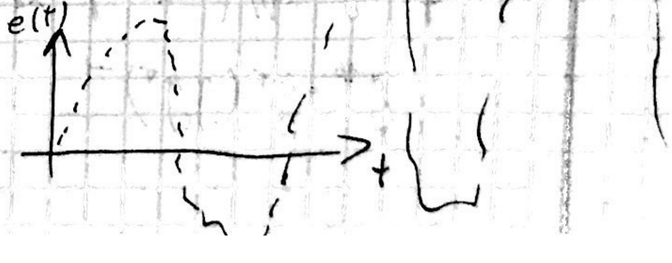
$$s_{1,2} = \frac{-k_d \pm \sqrt{\Delta}}{2}$$











$\Delta > 0 \quad \wedge \quad k_p > 0$

$\sqrt{\Delta} < k_d$

STABILNY ~~dla~~ $\boxed{k_p, k_d > 0}$

Klasyczny sterownik PD

ZADANIE ŚLEDZENIA TRAJEKTORII EFEKTORA

$x_d(\cdot)$

$$Q(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q) = u$$

$$x = k(q)$$

→ Metoda pośrednia

$$x_d(\cdot) \rightarrow q_d(\cdot) \rightarrow \text{przy pomocy CT}$$

$$e(t) = x(t) - x_d(t)$$

$$\dot{e} = -\gamma e \quad , \gamma > 0$$

$$e(t) \xrightarrow{t \to \infty} 0$$

$$\dot{e} = \dot{x} - \dot{x}_d \qquad \dot{x} = J(q)\dot{q}$$

$$J(q)\dot{q} - \dot{x}_d = -\gamma(k(q) - x_d)$$

$q(0)$ - znane

$$\dot{q} = J^{\#}(q)\left(\dot{x}_d + \gamma(x_d - k(q))\right)$$

$$J^{\#} = J^T(JJ^T)^{-1}$$

Wyznaczamy $q_d(\cdot)$

→ Metoda bezpośrednia

$$x = k(q) \quad \rightarrow \quad \dot{x} = J(q)\dot{q}$$

$$e(t) = x(t) - x_d(t)$$

$$\ddot{x} = \dot{J}(q)\dot{q} + J(q)\ddot{q}$$

$$\ddot{q} = Q^{-1}(u - C\dot{q} - D)$$

$$\ddot{x} = \dot{J}(q)\dot{q} + J(q)\cdot Q^{-1}(u - C\dot{q} - D)$$

$$= P(q, \dot{q}) + R(q)u = v$$

↓ nowe sterowanie

$$\ddot{x} = v$$

$$v = \ddot{x}_d - K_d(\dot{x} - \dot{x}_d) - K_p(x - x_d)$$

$$\ddot{e} + K_d\dot{e} + K_p e = 0$$